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ADP013894

TITLE: Fractional Cylindrical Functions Implementation for  
Electromagnetic Waves Scattering Analysis

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TITLE: 2002 International Conference on Mathematical Methods in  
Electromagnetic Theory [MMET 02]. Volume 2

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## FRACTIONAL CYLINDRICAL FUNCTIONS IMPLEMENTATION FOR ELECTROMAGNETIC WAVES SCATTERING ANALYSIS

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### ABSTRACT

An extension of a boundary integral method without using Green's function for solving electromagnetic boundary-value problems in layered media is presented. The generalization of the previous applications of that method to the case under consideration is achieved by using fractional cylindrical functions as the testing functions, which account for spatial inhomogeneity of the ambient medium. To illustrate advantages of such approach, numerical analyses are presented for 2D scattering problem involving penetrable cylinder of elliptical cross-section shape.

### INTRODUCTION

Employment of fractional cylindrical functions for analysis of electromagnetic wave scattering in the presence of bodies with coordinate surfaces was offered in [1,2]. But in our opinion, such functions can be used to solve many others problems of modern electromagnetic theory. Therefore, the attempt of expanding implementation area of such functions was undertaken. Two-dimensional scattering problem for homogeneous cylinders of arbitrary cross-section shape embedded in a plane layer is considered.

### PROBLEM STATEMENT AND BASIC EQUATIONS

Let suppose that as it shown in Fig.1 a permeable cylinder of arbitrary cross section  $S_p$  is situated in one layer of the three-layer structure. Introduce a coordinate system XYZ and suppose, that the impress sources  $f$  of monochromatic ( $\sim \exp(-i\omega t)$ ) wave and layers boundaries belongs to surrounding domain  $S_c$ . Cylinder formative is parallel to axis OX. The medium inside inclusion is described by material parameter  $\varepsilon_p(\vec{r})$  and

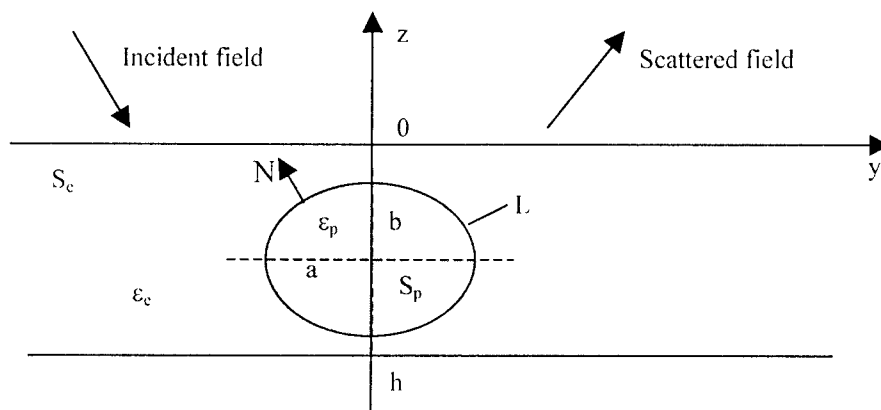


Fig. 1. Geometry of the problem.

wave number  $k_p(\vec{r})$ , and outside of inclusion -  $\varepsilon_e(\vec{r})$  and  $k(\vec{r})$  correspondingly.

Fields amplitudes outside and inside of inclusion satisfies to such equations:

$$\left[ \varepsilon(\vec{r}) \nabla_{\perp} \cdot \frac{1}{\varepsilon(\vec{r})} \nabla_{\perp} + k^2(\vec{r}) \right] \begin{Bmatrix} u_e(\vec{r}) \\ u_p(\vec{r}) \end{Bmatrix} = \begin{cases} \varepsilon(\vec{r}) \cdot f(\vec{r}), & (\vec{r} \in S_e) \\ 0, & (\vec{r} \in S_p) \end{cases} \quad (1)$$

(where  $\vec{r} = (0, x, y)$ ,  $\nabla_{\perp} = \left(0, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ ,  $f(\vec{r})$ -source density), and to boundary conditions:

$$u_e = u_p; \quad \frac{1}{\varepsilon} \frac{\partial u_e}{\partial N} = \frac{1}{\varepsilon_p} \frac{\partial u_p}{\partial N}; \quad (\vec{r} \in L). \quad (2)$$

They also satisfy continuity conditions for functions  $u_e$ ,  $1/\varepsilon$ ,  $\partial u_e / \partial N$ ,  $u_p$ ,  $1/\varepsilon$ ,  $\partial u_p / \partial N$  on structural boundaries in  $S_e, S_p$  and  $u_e$ , and satisfies radiation condition when  $r \rightarrow +\infty$ .  $\vec{N}$  - normal to  $L$  - contour of the scatterer, directed to  $S_e$ . In the case of excitation by vertically polarized wave ( $\vec{H} = \{H_x, 0, 0\}$ ,  $\vec{E} = \{0, E_y, E_z\}$ ), it is suitable to choose an x- components of magnetic field vectors as a functions  $u_e, u_p$ . As a result of manipulations described in [3], one can obtain following formulae for the scattered field:

$$u_{sc} = \int_L \frac{1}{\varepsilon(\vec{r}_L')} \left[ G(\vec{r}, \vec{r}_L') \frac{\partial u_e(\vec{r}_L')}{\partial N'} - u_e(\vec{r}_L') \frac{\partial G(\vec{r}, \vec{r}_L')}{\partial N'} \right] dL'; \quad (\vec{r} \in S_e), \quad (4)$$

where  $G(\vec{r}, \vec{r}')$ - Green's function of a regular problem. We'll represent an unknown functions  $u_e$  and  $\partial u_e / \partial N$  as an expansions in terms of functions  $\xi_{\sigma}$  and  $\partial \xi_{\sigma} / \partial N$ :

$$u_e(\vec{r}) = \sum_{\sigma} \alpha_{\sigma} \xi_{\sigma}(\vec{r}); \quad \partial u_e(\vec{r}) / \partial N = \sum_{\sigma} \beta_{\sigma} \partial \xi_{\sigma}(\vec{r}) / \partial N. \quad (6)$$

Unlike previous papers, we'll choose the function  $\xi_{\sigma}(\vec{r})$  in a following way:

$$\xi_{\sigma}(\vec{r}) = \begin{cases} f_m(r) \cos(m\varphi) & \sigma = (m, e), m = 0, 1, 2, \dots \\ f_m(r) \sin(m\varphi) & \sigma = (m, o), m = 1, 2, \dots \end{cases}; \quad \varphi = \arg(\vec{r}), \quad (8)$$

where  $f_m(r) = J_{m+1}(r) / J_m(r)$ ,  $J_m(r)$  - Bessel functions of order  $m$ . We'll compute the wave functions using the fractional cylindrical ones, as it was shown in [3].

A possibility of fractional cylindrical functions application to the considered problem justified by presence of cylindrical inclusion with sufficiently smooth contour of cross-section. Unlike the results obtained in [2], the discussed problem is complicated by presence of dielectric layer and cylinder with sufficiently arbitrary cross-section.

## DESCRIPTION OF NUMERICAL ALGORITHM AND ANALYSIS OF OBTAINED RESULTS

In this section some numerical results based both on the analytical techniques developed in previous papers and a new algorithm for evaluating scattering diagrams by means of fractional cylindrical functions are presented. The goal of numerical experiments was to determine effectiveness of the proposed approach. Computations performed for dielectric elliptic cylinders, immersed in homogeneous dielectric layer. They show that for cylinders with transversal dimensions less than wavelength application of fractional

cylindrical functions does not give noticeable advantage. This fact can be easily explained, if one take into account, that the basic advantages of fractional functions related to their smooth behavior attached to large index and argument values [2]. Also it's necessary to mention, that the most effective this procedure became in a case of circular cylinder.

In Fig. 2 scattering diagrams, calculated by means of Bessel and fractional cylindrical functions are shown. Dielectric permeabilities of cylinder, layer, upper and lower half-spaces are equal to  $\varepsilon_p = 1.2$ ,  $\varepsilon_s = 3.0$ ,  $\varepsilon_e = 1.0$ ,  $\varepsilon_c = 2.0$  accordingly. Cylinder's radius is  $2.0 \cdot \lambda$ , layer's thickness -  $h = 6 \cdot \lambda$ , an inclusion embedding depth -  $Z_p = 3 \cdot \lambda$ . Incident wave length -  $\lambda = 1.5$  m. One can see, that when Bessel functions are used, scattering diagrams became steady only when  $M \geq 30$  ( $M$  maximum order of the scattering matrix). On the other hand, for fractional cylindrical functions analogous results can be obtained when  $M=20$ , and additional increasing of number of terms in decomposition does not leads to any changes.

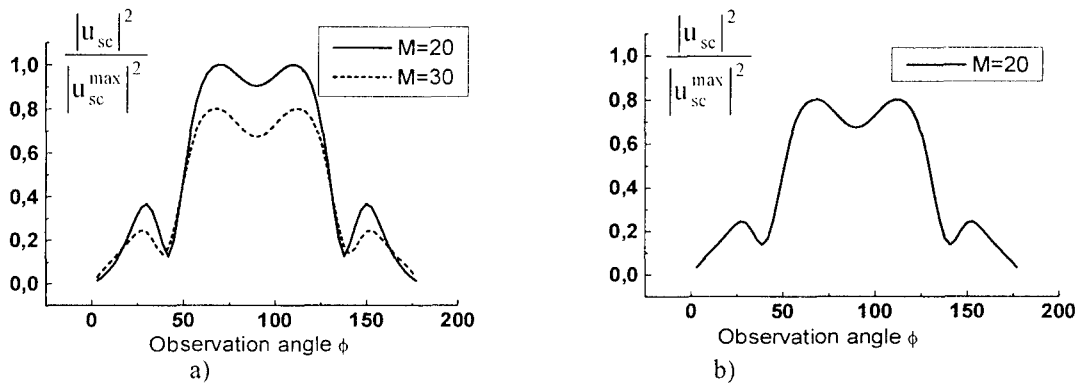


Fig.2. Scattered field diagram obtained for Bessel functions (a) and for fractional cylindrical functions (b).

## CONCLUSION

Obtained results show, that the fractional cylindrical functions within the framework of the null field method forms a successive base for solving a variety of scattering problems. Such problems in particular are essential elements in microwave and optical device design, nondestructive testing, remote sensing and thin film physics. The same technique can be applied to another diffraction problems (for example 3D problems) with only minor modifications.

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